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Electrical resistivity of thin wires at low temperatures: potassium

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Abstract. We present the results of a calculation of $\rho(T)$ for thin wires at low temperatures. The calculation includes the contributions arising from normal electron–electron scattering and from normal electron–phonon scattering, two terms that are absent for bulk samples. Applying the results to thin wires of potassium yields quantitative agreement with the hitherto unexplained $\rho(T)$ data of Rowlands, Yu, Zhao and co-workers.

1. Introduction

During the past decade, there has been considerable interest in the temperature-dependent part of the electrical resistivity $\rho(T)$ of thin wires and foils of the non-transition metals. Following the pioneering work of Sambles and Elsom (1980) and of Black (1980), many papers have appeared that discuss 'size-effect corrections' to $\rho(T)$ especially at low temperatures (Sambles *et al* 1981, 1982, Sambles and Preist 1982, Sambles and Elsom 1985, van der Maas *et al* 1981a, b, 1983, Caplin *et al* 1981, Stesmans 1982a, b, 1983, Tellier and Tosser 1982, Yu *et al* 1984, 1989, Zhao *et al* 1988, De Gennaro and Rettori 1984, 1985a, b, Kaveh and Wiser 1985a, b, Thummes *et al* 1985, Thummes and Kotzler 1985, Kuckhermann *et al* 1985, 1986, Kuckhermann and Mende 1986, Dimmich and Warkusz 1986, Moraga 1987, Gridin and Datars 1989, Gurzhi *et al* 1989a, b, Movshovitz and Wiser 1990a, b).

In spite of all these studies, certain sets of thin-wire $\rho(T)$ data remain unexplained. In this article, we present the results of a calculation of $\rho(T)$ at low temperatures, which includes the contributions of normal electron–electron scattering and normal electron–phonon scattering. These two scattering processes are *non-resistive* in the sense that they do not contribute to $\rho(T)$ for thick wires, but they do contribute for thin wires for which electron–surface scattering is important. Our calculated values for $\rho(T)$ agree quantitatively with the data for thin wires of potassium (Rowlands *et al* 1978, Yu *et al* 1984, Zhao *et al* 1988). Since the reported 'anomalies' for $\rho(T)$ for potassium are thus explained within the framework of non-resistive electron scattering (Black 1980, Wiser 1988), there is no need to invoke more exotic explanations, such as those based on charge density waves and electron–phason scattering (Bishop and Overhauser 1981, Bishop and Lawrence 1985) or on quantum localization (Farrell *et al* 1985).

In section 2, we present the recent resistivity data for thin wires of potassium. The theoretical ideas on which the resistivity calculation is based are presented in section 3.

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The method of 'electron dynamics' that we used to calculate $\rho(T)$ for non-resistive scattering is reviewed in section 4. In section 5, relevant previous work is discussed. The summary follows in section 6.

2. Resistivity data for thin wires of potassium

We begin by reviewing the temperature dependence of $\rho(T)$ for pure thick wires (no electron–surface scattering). Below about 1 K, the electron–phonon scattering term is negligible for potassium, and $\rho(T)$ is therefore dominated by the Umklapp electron–electron scattering term

$$\rho(T) = AT^2 \tag{1}$$

where the experimental value for potassium is $A_{\text{expt}} \simeq 0.26\ \text{p}\Omega\ \text{cm}\ \text{K}^{-2}$. The quadratic temperature dependence for $\rho(T)$ predicted by (1) was confirmed experimentally by Lee *et al* (1982) for 'thick' wires of potassium having a diameter larger than 1 mm.

The importance of the confirmation of (1) by Lee *et al* lies in the fact that when $\rho(T)$ was measured a few years earlier, anomalous behaviour was reported. Rowlands *et al* (1978) had found that the temperature dependence predicted by (1) was not observed for potassium, but rather $\rho_{\text{expt}}(T) \propto T^{1.5}$. We shall see that size-effect corrections provide the explanation for this deviation of $\rho(T)$ from the expected quadratic temperature dependence.

Attributing to electron–surface scattering the unexpected temperature dependence found for $\rho(T)$ may seem surprising at first thought, for the following reason. The diameter of the potassium wires measured by Rowlands *et al* was $d = 0.79$ mm, which is about three times the electron mean free path λ, and it has traditionally been thought that electron–surface scattering is not important for $\rho(T)$ unless the wire is so thin that $d < \lambda$. The reason for the relatively large size effects observed for $\rho(T)$ in the present case is that electron–surface scattering does *not* contribute to $\rho(T)$ by altering the AT^2 term in (1). That would indeed require wires so thin that $d \ll \lambda$. Rather, electron–surface scattering introduces two new contributions to $\rho(T)$, which are completely *absent* for thick wires. The novel feature of our calculation is the inclusion of both these non-resistive electron scattering processes, which together lead to such an important effect on $\rho(T)$ at low temperatures even for relatively thick wires.

The Michigan State University group have carried out extensive resistivity measurements below 1 K for thin wires of potassium (Yu *et al* 1984, Zhao *et al* 1988). Their data for $\text{d}\rho(T)/\text{d}T$ for wires of various diameters are displayed in figure 1. The broken straight line passing through the origin represents the thick-wire results that conform to equation (1). The theoretical curves for the thin wires will be discussed in section 3. Here, we concentrate on the data points measured for thin wires, which were taken directly from Zhao *et al* (1988). The full symbols (indicating wires prepared in a He atmosphere) are from their figure 1(*a*), and the open symbols (indicating a wire prepared in an Ar atmosphere) are from their figure 2. We note that the gas used in the preparation of the wire does not seem to affect these data. We have not included the results for the thinnest measured wires, for which $d \simeq 0.1$ mm, because of the metallurgical problems (discussed in detail by Zhao *et al* 1988) which prevent an accurate characterization of the sample parameters.

It is seen from figure 1 that for thin wires, significant deviations from equation (1) are observed. Not only does $\text{d}\rho/\text{d}T$ become progressively smaller in magnitude as the

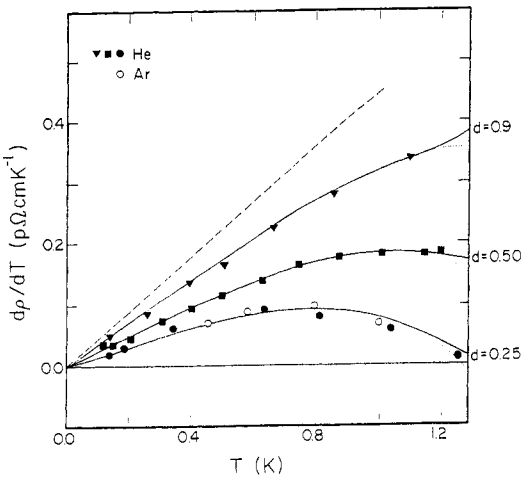

Figure 1. Temperature dependence of $d\rho(T)/dT$ for thin wires of potassium (values of d are in mm). The symbols represent the data of Zhao *et al* (1988) for wires having an electron–impurity scattering mean free path of $\lambda_i = 0.19$ mm. The full symbols represent wires prepared in a He atmosphere, whereas the open symbols represent a wire prepared in an Ar atmosphere. The full curves give the calculated values. The dotted curves for $T > 1.1$ K represent the values calculated without including the Umklapp electron–phonon scattering term. For comparison, the broken straight line gives the experimental values for a thick wire.

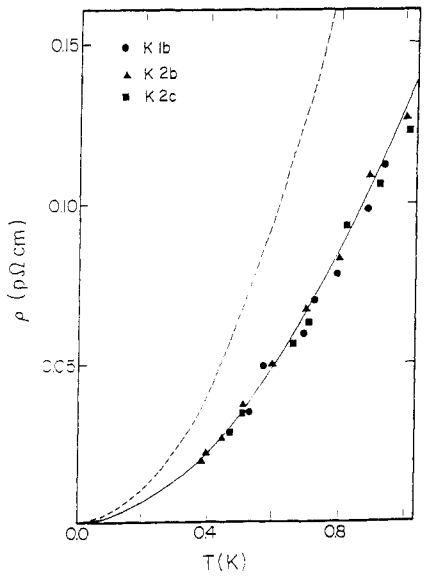

Figure 2. Temperature dependence of $\rho(T)$ for the indicated wires of Rowlands *et al* (1978). All three wires had the same diameter (0.79 mm) and the same residual resistivity (1.5 nΩ cm). The full curve gives the calculated values. For comparison, the broken quadratic curve gives the experimental values for a thick wire.

wire is thinner, but its temperature dependence also changes. Note in particular that even for the relatively thick wire having $d = 0.9$ mm, deviations from straight-line behaviour are clearly seen. As Yu *et al* (1984) themselves pointed out, the data for the $d = 0.9$ mm wire are reminiscent of the results reported earlier by Rowlands *et al* (1978) for wires having $d \simeq 0.8$ mm.

In figure 2, we present the data for $\rho(T)$ measured by Rowlands *et al*. (The theoretical curve will be discussed in section 3.) The symbols represent the experimental values for the three annealed wires that had the same residual resistivity ($\rho_0 = 1.5$ nΩ cm). Since all these wires also had the same diameter ($d = 0.79$ mm), they should yield the same values of $\rho(T)$—as indeed they do. For comparison, we include the broken curve, which gives the quadratic temperature dependence characteristic of thick wires, $\rho(T) = AT^2$, with the value of A taken from experiment.

What was most unexpected about these data is the magnitude of the size effect. It is seen that even for wires as thick as 0.8 mm in diameter, the size effect is so large that $\rho(T)$ is reduced to half its value.

3. Theoretical basis

The theoretical curves in figures 1 and 2 are based on the following ideas. For a thick wire of a non-transition metal, the contribution to $\rho(T)$ arising from normal electron–

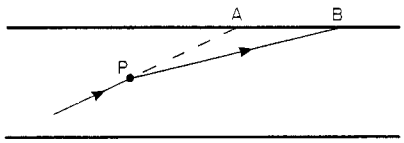

Figure 3. An electron undergoes a non-resistive collision at point P, which causes the electron to strike the surface of the thin wire at point B, rather than at point A.

electron scattering (NEES) vanishes because the total electron momentum in conserved at each NEES collision. For a thick wire of an alkali metal at low temperatures, the contribution to $\rho(T)$ arising from normal electron–phonon scattering (NEPS) vanishes because of phonon drag (Kaveh and Wiser 1972). However, for a thin wire, both NEES and NEPS *do* contribute to $\rho(T)$ by altering the direction of the electron trajectory.

Figure 3 illustrates the physical principle. An electron undergoes a non-resistive collision at point P. This collision alters the direction of the trajectory, causing the electron to strike the surface of the wire at the more distant point B, rather than at point A. The electron mean free path is thereby lengthened (PB > PA) and hence $\rho(T)$ is *decreased*. Alternatively, the point B could be nearer to point P than is point A (PB < PA). In the latter case, the electron mean free path is shortened and $\rho(T)$ is *increased*. The net effect of non-resistive NEES and NEPS on $\rho(T)$ is, of course, determined by which of these events dominates when one considers all possible electron trajectories.

The role of impurity scattering is very important, because electron–impurity scattering causes the electron to lose all memory of its previous trajectory. Therefore, the effect of non-resistive electron scattering illustrated in figure 3 does not occur if an electron–impurity scattering event takes place anywhere along the trajectory. Indeed, this is the reason that the discussion applies only to thin wires, by which one means a wire whose diameter is comparable to the mean free path for electron–impurity scattering.

We have calculated $\rho(T)$ using the method of 'electron dynamics', in which one follows the trajectory of each electron in the wire and calculates its mean free path between resistive collisions. Suitable averaging over all the electrons on the Fermi surface then yields $\rho(T)$. This method was introduced by Chambers (1950) in his pioneering study of electron conduction in thin wires. Chambers of course considered only *resistive* electron scattering. We have extended the chambers method to *non-resistive* electron scattering events to obtain the contributions of NEES and NEPS to $\rho(T)$ for thin wires at low temperatures.

An important parameter in the calculation relates to the roughness of the wire surface and determines the degree of specularity of electron–surface scattering. This parameter is usually denoted by p, where $p = 1$ means specular electron–surface scattering and $p = 0$ means diffuse electron–surface scattering. Rather than taking $p = 0$, as is often done, we found that it is important to use the approach of Soffer (1967), which takes account of the fact that the value of p depends on the angle at which the electron strikes the wire surface. The Soffer expression for this surface-roughness parameter is

$$p(\Theta) = \exp(-(4\pi\alpha\cos\Theta)^2) \tag{2}$$

where Θ is the angle between the electron trajectory and the normal to the surface of the wire, and α is the ratio of the root-mean-square surface roughness to the electron de Broglie wavelength. One expects α to be of order unity; we used $\alpha = 2$ for each wire. This reasonable value gave a good fit to the data, as shown by the full curves in figures 1 and 2.

Table 1. Values for the diameter d and the ratio λ_i/d for the wires that appear in figures 1 and 2. The calculated values of γ and β for each wire are also given, where γ and β are defined in equation (3).

Experiment	d (mm)	λ_i/d	γ (pΩ cm K^{-2})	β (pΩ cm K^{-5})
Zhao *et al* (1988)	0.9	0.21	0.082	0.0074
	0.5	0.38	0.138	0.0120
	0.25	0.76	0.182	0.0152
Rowlands *et al* (1978)	0.79	0.30	0.12	0.011

For the thin wires of potassium for which measurements were made, our calculation (to be described in the next section) yields that both NEPS and NEES make a *negative* contribution to $\rho(T)$:

$$\rho_{\mathrm{NEES}}(T) = -\gamma T^2 \qquad \rho_{\mathrm{NEPS}}(T) = -\beta T^5 \tag{3}$$

where the values of both γ and β depend on the ratio λ_i/d. The electron–impurity scattering mean free path λ_i has the same value of 0.19 mm for all the wires in figure 1 since they all correspond to potassium having a bulk RRR of 7300 (Zhao *et al* 1988).

The calculated values of γ and β are listed in table 1 for each of the wires in figures 1 and 2. These values, when added to the Umklapp electron–electron scattering term given in equation (2), yield the theoretical curves plotted in the figures.

We have also included the small Umklapp electron–phonon scattering term, using the standard expression $\rho_{\mathrm{UEPS}}(T) = BT\,\mathrm{e}^{-20/T}$, where $B = 8000$ pΩ cm K^{-1} for each wire (van Kempen *et al* 1981). This term begins to contribute only above 1.1 K. The dotted curves in figure 1 for $T > 1.1$ K give the values calculated without this term.

For each of the three wires of Rowlands *et al* shown in figure 2, we assumed $\lambda_i = 0.24$ mm, which corresponds to a somewhat smaller value of the electron–impurity scattering resistivity than the measured residual resistivity. For a basis for this assumption, see Yu *et al* (1989). It is of course the contribution due to NEPS, of the form $-\beta T^5$, that accounts for the lack of quadratic temperature dependence in the data of Rowlands *et al*. Although the NEPS term is quite small—being only 8% of the quadratic electron–electron scattering term at 1 K—because of its negative sign, it suffices to make $\rho(T)$ fit better to $T^{1.5}$ than T^2.

The agreement between theory and experiment is evident from figures 1 and 2. In particular, the calculation reproduces the two principal features of the data: the deviations from a quadratic temperature dependence for $\rho(T)$, and the much smaller values of $\rho(T)$ than those measured for thick wires.

4. Method of 'electron dynamics'

Our extension of the Chambers method of 'electron dynamics' to the case of non-resistive NEES and NEPS is described in detail in a series of articles (Movshovitz and Wiser 1990a, b, c). Nevertheless, it is useful to summarize here the basic principles of the method.

To determine the change in the electron mean free path due to non-resistive scattering, one calculates the electron mean free path with and without the non-resistive

scattering, and then takes the difference between these two quantities. We shall discuss each of these quantities in turn.

4.1. Resistive scattering

Consider an electron with mean free path λ_i for electron–impurity scattering. The probability $P(r)$ for the electron to travel a distance r without being scattered is

$$P(r) = e^{-r/\lambda_i} \tag{4}$$

and the probability dQ for an electron–impurity collision to occur within the infinitesimal distance dr at a distance r from the origin of the electron trajectory is

$$dQ(r) = (dr/\lambda_i)\, e^{-r/\lambda_i}. \tag{5}$$

The average distance travelled by the electron before being scattered, Λ_0, is then given by

$$\Lambda_0 = \int r\, dQ. \tag{6}$$

In a bulk sample, the limits on the integral are 0 and ∞, leading to $\Lambda_0 = \lambda_i$, as required. However, if the wire has a finite diameter, then one obtains

$$\Lambda_0 = \lambda_i(1 - e^{-R/\lambda_i}) \tag{7}$$

where R is the distance to the wire surface in the direction of the electron trajectory. The second term in (7) gives the reduction in Λ_0 due to the additional resistive collisions that take place at the surface of the wire.

The relevant geometry is illustrated in figure 4(*a*). The electron begins its journey at the point 0 and travels in the direction θ, φ. In this direction, the distance to the wire surface is $R(\theta, \varphi)$. To obtain the average mean free path for the electrons, one integrates $\Lambda_0(\theta, \varphi)$ of (7) over all possible values of θ, φ and all possible origins 0 (Movshovitz and Wiser 1990a, b).

The expression for Λ_0 given in (7) assumes that electron–surface scattering is totally diffusive. However, we want to take account of the reality that electron–surface scattering is, in fact, partially specular. This is done by introducing the specularity parameter p, where $p = 0$ and $p = 1$ denote diffuse and specular electron–surface scattering, respectively. Incorporating a non-zero value for the parameter p changes (7) into

$$\Lambda_0 = \lambda_i[1 - (1 - p)\, e^{-R/\lambda_i}/(1 - pe^{-R'/\lambda_i})] \tag{8}$$

where R' is a distance related to R (for details, see Chambers 1950). It is readily seen from (8) that for $p = 0$, one recovers (7), whereas for $p = 1$, $\Lambda_0 = \lambda_i$ and electron–surface scattering has no effect at all.

An important feature of our calculation is that we take account of the fact that the value of $p(\Theta)$ depends on the angle Θ between the electron trajectory and the normal to the wire surface. The expression for $p(\Theta)$ has been given in equation (2).

4.2. Non-resistive scattering

The presence of non-resistive (NR) scattering events (either NEES or NEPS) leads to two additional contributions to the electron mean free path, now denoted Λ_{NR}. The first

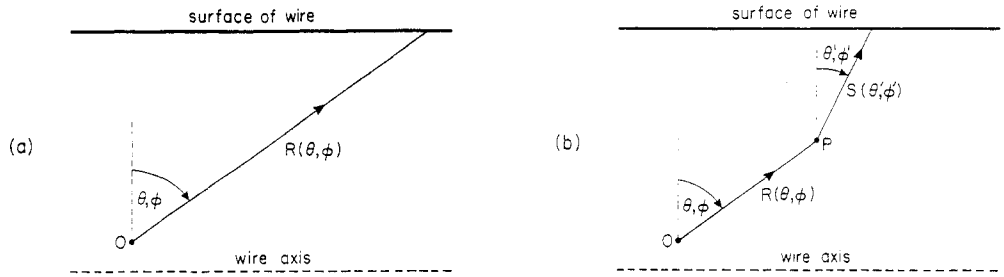

Figure 4. Electron trajectories in the direction θ, φ starting at the point 0. (*a*) Non-resistive scattering does not occur. Then, the distance along this trajectory to the surface of the wire is $R(\theta, \varphi)$. (*b*) Non-resistive scattering does occur. The electron travels a distance $R(\theta, \varphi)$ and then undergoes a non-resistive collision at the point P, which alters the direction of its trajectory to θ', φ'. The distance along this new direction to the surface of the wire is $S(\theta', \varphi')$.

contribution arises from the fact that the electron mean free path without surface scattering, denoted λ, is shortened by NR scattering. Thus,

$$1/\lambda = 1/\lambda_i + 1/\lambda_{NR} \tag{9}$$

where λ_{NR} is the electron mean free path due only to NR scattering events, and as before, λ_i is due to electron–impurity scattering. This implies that to obtain Λ_{NR}, one must replace λ_i by λ in the appropriate places in the analysis of Λ_0 given in the previous subsection.

Since $\lambda_{NR} \gg \lambda_i$, one expands λ to obtain

$$\lambda \simeq \lambda_i - \lambda_i^2/\lambda_{NR}. \tag{10}$$

The second term in λ then leads to a contribution to Λ_{NR} that is inversely proportional to λ_{NR}.

The second additional contribution to Λ_{NR} arises from the type of electron trajectory that is illustrated in figure 4(*b*). As in figure 4(*a*), the electron begins its journey at point 0, travelling in the direction θ, φ. However, after traversing a distance $R(\theta, \varphi)$ and reaching the point P, the electron undergoes a NR collision which alters the direction of its trajectory to θ', φ'. In this new direction, the distance to the surface of the wire is $S(\theta', \varphi')$. Thus, the total distance from the initial point 0 to the wire surface is $R + S$.

The type of trajectory shown in figure 4(*b*) will occur only if a NR collision takes place at some point P, and hence its probability depends on the magnitude of $1/\lambda_{NR}$. Thus, we obtain a second contribution to Λ_{NR} that is inversely proportional to λ_{NR}. The analysis of these trajectories that involve NR scattering is somewhat more complicated than those involving only resistive scattering; the details are given in Movshovitz and Wiser (1990a, b).

Figure 4(*b*) depicts the two segments of the electron trajectory, R and S, as if they were both lying in the same plane as the wire axis. This is, of course, not generally the case. Indeed, we found that it is very important to take into account the three-dimensional geometry of the scattering processes.

Having calculated Λ_{NR}, one can express the change $\Delta\Lambda_{NR}$ in the electron mean free path due to NR scattering

$$\Delta\Lambda_{NR} = \Lambda_{NR} - \Lambda_0 \tag{11}$$

as a contribution to $\rho(T)$,

$$\rho_{NR}(T) = -C\,\Delta\Lambda_{NR}/\Lambda_0^2 \tag{12}$$

where $C \equiv mv_F/ne^2$. The quantity Λ_0 appearing in (11) and (12) denotes the value obtained from $\Lambda_0(\theta, \varphi)$ in (8) after integrating over all possible values of θ, φ; similar remarks apply to Λ_{NR}. Combining (12) with the result

$$\Delta\Lambda_{NR} \propto 1/\lambda_{NR} \tag{13}$$

leads to

$$\rho_{NR} \propto 1/\lambda_{NR} \tag{14}$$

where λ_{NR} depends on the temperature.

4.3. NEES and NEPS

The difference between NEES and NEPS lies in the following. For NEES, one may assume that the probability for non-resistive scattering is isotropic. The work of Black (1980) has shown that this is a good approximation. Therefore, after the NEES collision at point P, all values of θ', φ' are equally likely. However, for NEPS at low temperatures, only small-angle scattering events occur, greatly restricting the possible values of θ', φ'. This is the fundamental difference between NEES and NEPS, with regard to the calculation of $\rho_{NEES}(T)$ and $\rho_{NEPS}(T)$.

Applying equation (14) to NEES implies that the temperature dependence of $\rho_{NEES}(T)$ arises from the temperature dependence of $\lambda_{NEES}(T)$.

$$\rho_{NEES}(T) \propto 1/\lambda_{NEES}(T) \propto T^2 \tag{15}$$

leading to the result quoted above in equation (3).

For NEPS, the analysis is somewhat more complicated because the inverse cubic temperature dependence of $\lambda_{NEPS}(T)$ is not the only temperature dependence of $\rho_{NEPS}(T)$. Kaveh and Wiser (1985b) have shown that at low temperatures, there is an additional T^2-dependence that arises from the severe restriction of the possible values of θ', φ' after a small-angle NEPS collision at the point P in figure 4(*b*). Combining these two temperature dependences yields

$$\rho_{NEPS}(T) \propto T^2/\lambda_{NEPS}(T) \propto T^5 \tag{16}$$

leading to the result quoted above in equation (3).

The constants of proportionality in (15) and in (16), denoted by γ and β in equation (3), have been calculated (Movshovitz and Wiser 1990b, c). The results for the thin wires in figures 1 and 2 are listed in table 1, yielding the theoretical curves plotted in the figures.

5. Previous work

The idea that non-resistive NEPS contributes to $\rho(T)$ for thin wires of the alkali metals at low temperatures was first proposed by Kaveh and Wiser (1985b). The idea that

non-resistive NEES makes a negative contribution to $\rho(T)$ because of the presence of impurities in the wire was first proposed by De Gennaro and Rettori (1984). We subsequently criticized the De Gennaro–Rettori paper on the basis of a Monte Carlo calculation of NEES (Movshovitz and Wiser 1987), but our present work shows that this criticism was incorrect.

An interesting discussion of $\rho(T)$ for thin wires has recently been given by Gurzhi *et al* (1989a, b). These workers used the Boltzmann equation to calculate $\rho(T)$, analyzing in detail the collision integral for non-resistive NEES and NEPS. However, their intricate analysis did not yield numerical values for $\rho(T)$ that can be compared with the experimental values. Moreover, like all earlier workers, they assumed that electron–surface scattering is diffusive, an approximation that is not adequate to account quantitatively for the experimental data.

6. Summary

We have reported the results of a calculation to $\rho(T)$ in which the contributions of both NEES and NEPS are determined within a unified theoretical framework. Moreover, we have taken explicit account of the important angular dependence of the surface-roughness parameter $p(\Theta)$. The theoretical curves of figures 1 and 2 show that the resistivity data for thin wires of potassium can be accounted for quantitatively—both their magnitude and their temperature dependence.

Acknowledgments

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